Prototype Solver

As a first step in developing an operational phase solver for ALMA we implemented some of the necessary algorithms in MATLAB, and applied them to the same data as used in our coherence studies above. We used non-weighted linear least squares to estimate a phase offset for $(n-1)$ antennas to a reference antenna, independently for each correlator dump (0.96s). The normal equations and partial derivatives have a simple form due to the nature of the data, for which we used the phase differences between antenna pairs. Thus the observable is simply the angle of the complex visibility; between antennas *k* and *l* it is O_{kl} = ang (*visibility*$_{kl}$). The computed model that is implemented is $C_{kl} = \phi_l - \phi_k$, where ϕ_l and ϕ_k are the model phases at antennas l and k. If we define $\bar{\phi}$ to be the column vector of model phases $\bar{\phi} = [\phi_1 .. \phi_n]^T$, then

$A\bar{\phi} = B$, yielding the solution $\bar{\phi} = A^{-1}B$, with

$$A_{ij} = \sum_{k,l} \frac{\partial C_{kl}}{\partial \Phi_i} \cdot \frac{\partial C_{kl}}{\partial \phi_j} \qquad B_j = \sum_{k,l} \frac{\partial C_{kl}}{\partial \phi_j} \cdot (O_{kl} - C_{kl}) \text{ and } \frac{\partial C_{kl}}{\partial \phi_i} = \delta_{il} - \delta_{ik}$$

where *i* and *j* are antenna indices running from 2..n. The double sums are over all n(n-1)/2 visibilities, which are used to determine the n-1 phase offsets. In this work each correlator dump yielded a separate solution, but for weaker sources some integration over time may be necessary. Since all antenna weights were treated as unity, the A matrix had a simple, constant form of 15's along the diagonal and -1 elsewhere, which allowed it to be inverted once, at the outset. Note that the use of weighting factors (relative to the reference antenna), would necessitate the inversion of A whenever the antenna weights change, though this is still a minor computational burden.

In this work the phases are determined relative to an arbitrary reference antenna, but it is important for VLBI to then adjust the phases to be relative to the array average. This "common mode" phase doesn't have any effect on the coherence of the ALMA phased sum, but it is significant when correlating with other VLBI antennas. The premise is that the mean of the phases of all antennas is better behaved (with respect to atmospheric and instrumental fluctuations) than a single antenna. This was born out by the observation that the rms over time of the antenna phases relative to a pool mean was lower than the rms with one antenna fixed, by amounts in the range of 10-25 %.

Since voltage data from the antennas were not available, we tested the solution by counter-rotating the complex visibilities from each antenna pair. A plot of the rotated visibilities can be seen in Figure 1. Since this problem is linear in the correction phases, there was no iteration necessary in the fitter. Also, it is necessary to start with some initial guess at the phases, and to counter-rotate the data prior to the fit with the a priori values. To the least squares fitter a phase of 20° is not the same as a phase of 380°, since the mean phase is determining the fit result. The counter-rotation can be approximate, with no effect on the end result. An automated system can start with phases relative to a single antenna to start, and then track the phases over time for use as a subsequent a priori.

This model solver can now be used to fairly simply explore a number of effects, e.g.:

- time lags in the application of the extracted phases
- poorer snr (by narrowing the bandwidth)
- smoothing of phase solutions over time and predictive filtering
- the effect of corrupted antennas
- etc.

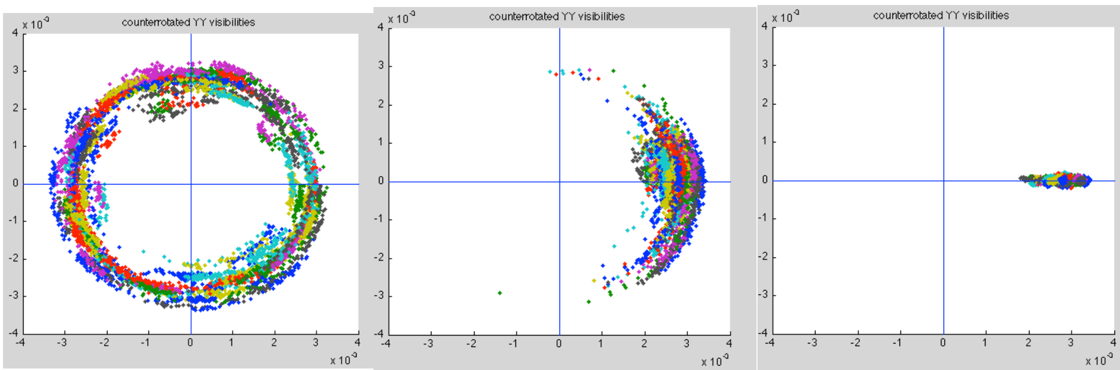

Figure 1 WVR-corrected YY fringe visibilities for all 120 baselines (color-coded by baseline), for 63 accumulation periods, averaged over a 1.8 GHz band. The left panel shows the complex visibilities direct from the correlator, after scaling via autocorrelation values. The central panel shows data that have been counter-rotated using antenna phases that were derived at the center of the time span. The right panel has had all baselines counter-rotated by a best-fit set of phases for each accumulation period. The coherent sum of the visibility vectors had a magnitude of 99.99% as large as the sum of the incoherent magnitudes.