Proposed Phasing Algorithm

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These are my notes based on the discussion we had at Haystack earlier this month.

1 Basic algorithm

At a given time t, the correlator baseline data are channel averaged in each baseband. From these data, the baseline phases are computed; named $\varphi_{\rm OBS}(t)$.

From the source model at large angular scales, the baseline model phases $\varphi_{\rm MOD}(t)$ are also computed, using a table sent by TelCal. These are set to zero if the model is a point source.

The observed phases are assumed to be the model phases affected by atmospheric and instrumental errors, by the correction phases, and by thermal noise:

$$\varphi_{\rm OBS}(t) = \varphi_{\rm MOD}(t) + \varphi_{\rm INS}(t) + \varphi_{\rm ATM}(t) - \varphi_{\rm C}(t) + \varphi_{\rm N}(t)$$

The goal is to phase the array for a point source in the field center, for which the model phases are zero; so the correction should as much as possible cancel the error terms:

$$\varphi_{\rm C}(t) = \varphi_{\rm INS}(t) + \varphi_{\rm ATM}(t)$$

We have to estimate the error terms from the correlator and WVR data. As the error and correction phases are actually antenna-based, one does at regular intervals an antenna-based solution:

$$\psi_{\rm OBS}(t) = L \cdot (\varphi_{\rm OBS}(t) - \varphi_{\rm MOD}(t))$$

L is a linear transformation which depends on the antenna weights and can be calculated before hand based on these weights. We note antenna-based phases as ψ and baseline based phases as φ; antenna phases are set to zero at the reference antenna. The transformation matrix L is calculated by TelCal (see below).

Then:

$$\psi_{\mathrm{OBS}}(t) = \psi_{\mathrm{INS}}(t) + \psi_{\mathrm{ATM}}(t) + \psi_{\mathrm{N}}(t) - \psi_{\mathrm{C}}(t)$$

The antenna-based noise $\psi_{\mathrm{N}}(t)$ is actually reduced by a factor $\sqrt{N}$ from the baseline-based noise. It is further reduced by getting a longer integration time, but the atmospheric term may be fluctuating on a time-scale of about one second. The instrumental term however is slowly varying and can be assumed constant on a time scale of tens of seconds.

The proposed scheme is then to split the correction into fast and slow terms:

$$\psi_{\mathrm{C}}(t) = \psi_{\mathrm{CF}}(t) + \psi_{\mathrm{CS}}(t)$$

and use two correction loops, of cycles t_f ($\sim$ 1s), and t_s ($\gtrsim$ 10s).

1. In the fast loop we use as a fast component the WVR correction calculated using the WVR data in the previous interval, and a constant slow component:

$$\psi_{\mathrm{C}}(t) = \psi_{\mathrm{WVR}}(t - t_f) + \psi_{\mathrm{CS}}(t_0)$$

   This is applied every t_f interval, until t_s is elapsed. The WVR corrections are calculated as in normal interferometry observations, except that the correction that would normally be applied to the reference antenna is subtracted from all the others.

2. At the end of t_s, we use the averaged observed data, as described above, to get the correction for the next interval $t_0 + t_s$.

$$< \psi_{\mathrm{OBS}}(t) >= (< \psi_{\mathrm{INS}}(t) > - \psi_{\mathrm{CS}}(t_0)) + (< \psi_{\mathrm{ATM}}(t) - \psi_{\mathrm{WVR}}(t) >)$$

   We will use:

$$\psi_{\mathrm{CS}}(t_0 + t_s) =< \psi_{\mathrm{OBS}}(t) > - \psi_{\mathrm{CS}}(t_0)$$

   thus including in the next slow term the change in instrumental correction and a possible slow systematic error in the WVR estimate of the atmospheric phase.

2 Specific cases

2.1 Very good weather

In that case the atmosphere phase fluctuations are very small and the WVR correction would only add noise to the correction. In that case it is better to set $\psi_{\rm CF}(t) = 0$ at all times, using only the slow correction loop.

2.2 Weak source

In that case the project source cannot be used to phase the array, and a nearby quasar is used, observed in a dedicated scan of duration t_s. That scan is only to estimate $\psi_{\rm CS}(t_0 + t_s)$ which will be used during the observing scans on the project source, until the next observation on the phasing calibrator.

3 Global WVR correction

The algorithm as described above does not apply any correction to the reference antenna, as the correction that would be applied to the reference antenna is subtracted from all the others. This is is the minimal correction to phase the array.

However the correction for the reference antenna should be stored with the VLBI data in order to be applied to the Alma station in the final correlation with the VLBI data from the other stations. This can be the most important use of WVR data in the case of very short baselines.

4 Antenna-based solution

In the determination of the linear transformation L we use a least squares solution. If the baseline data have weights that can be split into antenna-based weights w_i, the antenna-based solution for phases ψ_i can be expressed as:

$$\psi_i = \frac{1}{W}\left[\sum_{j,\ j\neq i} w_j \varphi_{ij} - \sum_{j,\ j\neq r} w_j \varphi_{rj}\right]$$

where W is the sum of antenna weights, and r is the index of the reference antenna. So the L matrix is essentially determined by the antenna weights, that may be assumed constant over a slow cycle ot the algorithm.

If we have do down-weigh long baselines where most of the source flux is resolved, then the weights cannot be split any more and the solution has to be derived numerically. But the weights should not change fast with time, and L should still be essentially constant over a slow cycle of the phasing algorithm.